



Compact Course Polynomial Optimization – Series 2

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Exercise 2.1

The field $\mathbb{R}(X)$ can be ordered in more than one way.

- a) Try to find orderings other than the one presented in the lecture that extend the ordering of \mathbb{R} .
- b) Can you describe all such orderings?

Exercise 2.2

Show the following: For a real closed field R , the set ΣR^2 is the unique ordering of R . Even more specifically, an element of R is non-negative if and only if it is a square x^2 with $x \in R$.

Exercise 2.3

Let $k \in \mathbb{N}$. Is the positive semidefinite cone \mathcal{S}_+^k semialgebraic? Is it basic semialgebraic?

Exercise 2.4

Find a quantifier-free formula equivalent to

$$F(p, q) := \exists x \left[-1 \leq x \leq 1, x^2 + px + q = 0 \right].$$

Draw a sketch of the respective semialgebraic set $S := \{(p, q) \in \mathbb{R}^2 : F(p, q) \text{ is true}\}$. Is it basic semialgebraic?