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DFG-Graduiertenkolleg
MATHEMATISCHE
KOMPLEXITÄTSREDUKTION

Compact Course Polynomial Optimization – Series 3

https://www.mathcore.ovgu.de/TEACHING/COMPACTCOURSES/2020opt.php

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Exercise 3.1

Show that the slice of

$$\bar{P}_{2,2}(\Delta) := \{(a,b,c) \in \mathbb{R}^3 \mid f := ax^2 + bxy + cy^2 \geq 0 \text{ on } \Delta\}$$

by the hyperplane $a+b+c=1$ is unbounded.

Exercise 3.2

Show the following: Let $X_1, \ldots, X_n, Y_1, \ldots, Y_n$ be indeterminates, and let E_+^n and E_-^n , respectively, be the set of all vectors $e \in \{-1,1\}^n$ with an even resp. odd number of entries equal to -1. Then

$$X_1 \cdot \ldots \cdot X_n \pm Y_1 \cdot \ldots \cdot Y_n = \frac{1}{2^{n-1}} \sum_{e \in E_\pm^n} \prod_{i=1}^n (X_i + e_i Y_i).$$

In particular, $X_1 \cdot \ldots \cdot X_n \pm Y_1 \cdot \ldots \cdot Y_n$ belong to the semiring generated by $X_1+Y_1, \ldots, X_n+Y_n, X_1-Y_1, \ldots, X_n-Y_n$.

Exercise 3.3

Show that if a polynomial f is positive on $\{a \geq 0\}$, then

$$(1+h)f = 1+g$$

holds for some $g, h \in \mathcal{P}(a)$.

Hint: Use the polynomial version of Farkas lemma: If $-1 \in \mathcal{P}(a)$, then every polynomial is in $\mathcal{P}(a)$.