Dr. Maximilian Merkert
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DFG-Graduiertenkolleg
MATHEMATISCHE
KOMPLEXITÄTSREDUKTION

Compact Course Polynomial Optimization – Series 4

https://www.mathcore.ovgu.de/TEACHING/COMPACTCOURSES/2020opt.php

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Exercise 4.1

Let $m, n \in \mathbb{N}$, let $K \subseteq \mathbb{R}^n$ and $L \subseteq \mathbb{R}^m$ be closed convex cones and let $A \in \mathbb{R}^{m\times n}, c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. Consider the problems

$$\alpha := \inf\{\langle c, x\rangle \mid x \in K,\ Ax - b \in L\}$$

and

$$\beta := \sup\{\langle y, b\rangle \mid y \in L^*,\ c - A^\top y \in K^*\}.$$

Show that the following hold:

(a) The problems satisfy weak duality.

(b) If there exists an x' with $x' \in \operatorname{int}(K)$ and $Ax - b \in \operatorname{int}(L)$, then strong duality holds.

(c) If there exists an y' with $y' \in \operatorname{int}(L^*)$ and $c - A^\top y \in \operatorname{int}(K^*)$, then strong duality holds.

Exercise 4.2

Show that for every $k \in \mathbb{N}$ one has $(\mathcal{S}^k_+)^* = \mathcal{S}^k_+$ (i.e., the cone $\mathcal{S}^k_+$ is self-dual).

Exercise 4.3

Formulate the problem of computing the largest eigenvalue of a symmetric matrix as an SDP.

Exercise 4.4

Consider the SDP

$$\inf\left\{x : \begin{pmatrix} 0 & x \\ x & y \end{pmatrix} \text{ psd},\ x \geq -1\right\}.$$

(a) What is the optimal value of this problem?

(b) What is the optimal value of its dual?